Hydropower Plants: Generating and Pumping Units
Series 2

GENERAL STUDY OF HYDRO POWERPLANT

The studied hydro-electric installation, whose layout is shown in Figure 1, is located in South America.

Figure 1: Layout of the hydro-electric installation

The rated discharge of the power plant Q is 50 m3 s-1. The gross head is considered as constant throughout the year and equal to 300 m. The value of the water density is ρ = 1'000 kg.m-3 and the gravity acceleration is g = 9.805 m.s^{-2}. The global efficiency of the power station (i.e. without considering the losses between B and I) is η = 0.91. For the purpose of simplification, the piping system is considered as a simplified penstock line, as illustrated in Figure 2. In this section, the only considered specific energy losses in the hydraulic circuit from B to I are the regular specific energy losses between III and II. Use the length of the penstock L = 5'400 m.

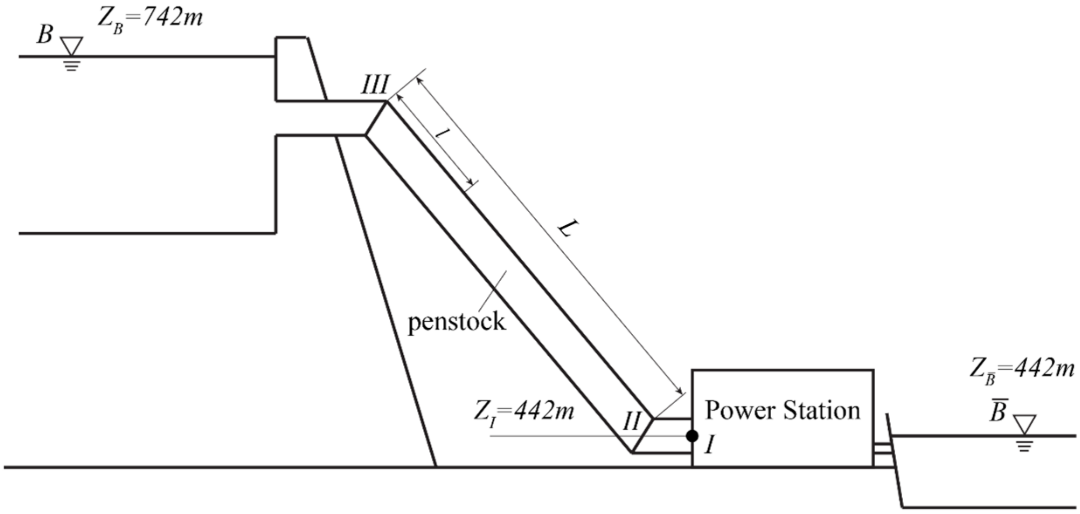

Figure 2: Simplified schematics of the hydro-electric installation

1) Express the potential power by using the relevant variables.

$$P_{potential} = \rho Q g \left(Z_B - Z_{\bar{B}} \right)$$

2) Express the distributed specific energy losses in the penstock $gH_{r\_penstock}$ as a function of the local loss coefficient λ, the length of the penstock L, the penstock diameter D, and the discharge Q.

$$gH_{r\_penstock} = \lambda \frac{L}{D} \frac{C^2}{2} = \lambda \frac{L}{D} \frac{\left(\frac{Q}{A} \right)^2}{2} = \lambda \frac{8L}{\pi^2 D^5} Q^2$$

3) Considering the distributed specific energy losses in the penstock, express the hydraulic power P_h at point *I*.

$$P_h = \rho Q \left(g \left(Z_B - Z_I \right) - gH_r \right) = \rho Q \left(g \left(Z_B - Z_I \right) - \lambda \frac{8L}{\pi^2 D^5} Q^2 \right)$$

4) The global efficiency η is the result of a combination of several phenomena which limit the use of the hydraulic power P_h by the hydraulic machine, thus reducing the final output power *P*. Provide an answer to the following questions:

   a. Is the transferred power P_t, also called extracted power, larger or smaller than the hydraulic power P_h? Justify your answer by giving a list of the phenomena that are responsible for this difference, and indicate which efficiency sub-terms η_i of the global efficiency η are involved.

   The extracted power is smaller than the hydraulic power due to two main factors:

   1. *The hydraulic machine is not ideal, i.e. there are some specific hydraulic energy losses that prevent the transfer of the whole specific energy contained in the water flow discharge to the turbine blades.*
   2. *Leakage flow losses reduce the actual flow discharge which is carrying the specific energy to the turbine.*

The corresponding efficiency sub-terms of the global efficiency η involved in this process are the following:

1. *The energetic efficiency η_e, representing the specific hydraulic energy losses E_{rb} and E_{rs}.*
2. *The leakage flow efficiency η_q, representing the losses of flow discharge.*

b. Is the mechanical power P_m larger or smaller than the output power P?

Why?

The mechanical power is larger than the output power since the bearing power losses prevent the transfer of the whole mechanical power, contained in the turbine in rotation, to the shaft connected to the generator.

c. To evaluate the electrical power which is actually provided to the grid, calculating the output power P is not enough. A further efficiency sub-term (which is however not included in the global efficiency η) must be considered. Why?

During the conversion from mechanical power to electrical power, some losses occur in the generator. The generator efficiency must also be taken into account to evaluate the actual power provided to the grid.

All the types of losses are illustrated in Figure 3, which is taken from Slide 5 in L2.

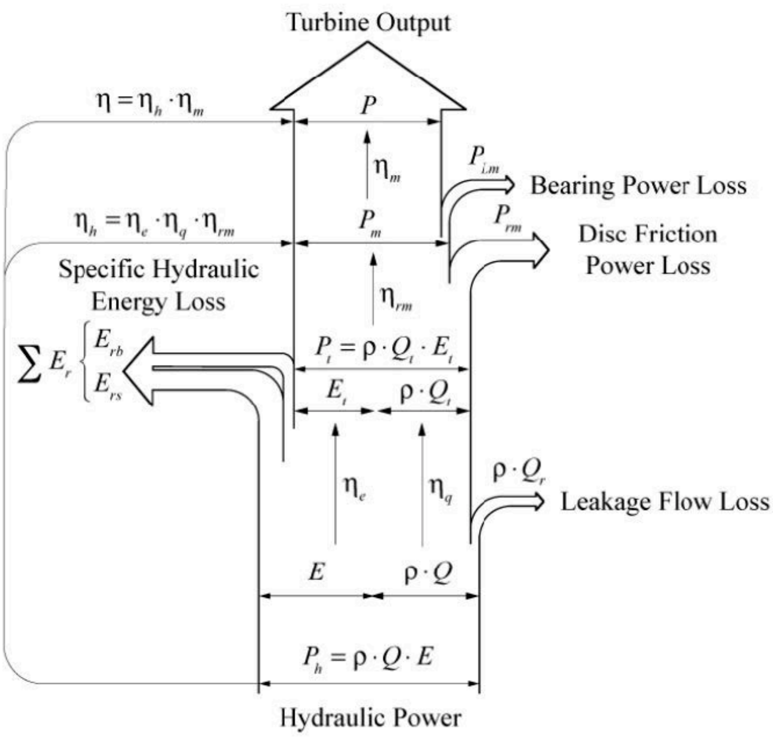

Figure 3: Losses in a turbine

5) Using the global efficiency η, express the output power P using the parameters introduced in questions 1) and 2).

$$P = \eta\rho Q\left(g\left(Z_B - Z_{\overline{B}}\right) - gH_r\right) = \eta\rho Q\left(g\left(Z_B - Z_{\overline{B}}\right) - \lambda\frac{8L}{\pi^2 D^5}Q^2\right)$$

6) Let's move now to the energy state of the flow discharge. Start by expressing the gauge pressure of a fluid in function of the absolute and the atmospheric pressures.

$$p_{gauge} = p_{absolute} - p_{atmosphere}$$

7) Performing an energy balance between the pressure, kinetic energy and potential energy components of the discharge flowing in the penstock, express the total gauge pressure $p_{tot}(l) = p_{gauge}(l) + \frac{1}{2}\rho u^2(l)$ as a function of L, l, Z_I and Z_{III}. Consider l as the coordinate which describes the longitudinal position in the penstock ($l = 0$ in *III*, and l = L in *II*), $p_{gauge}(l)$ the gauge pressure and $\frac{1}{2}\rho u^2(l)$ the dynamic pressure. Assume that the elevation in III is approximately the same as at the surface of the upper reservoir, $Z_{III} \cong Z_B$, and neglect the specific energy losses term.

Considering the energy balance between position l and B and the relation above, the following equation can be deduced;

$$p_{gauge}(l) + \frac{1}{2}\rho u^2(l) + \rho g\left(Z(l) - Z_{\overline{B}}\right) = p_{gauge,B} + \frac{1}{2}\rho u_B{}^2 + \rho g\left(Z_B - Z_{\overline{B}}\right)$$

Knowing, that $p_{gauge,B} = 0$ *because the absolute pressure in B is the atmospheric pressure, that* $u_B = 0$ *because the reservoir can be considered as perfectly still, and that* $Z_I = Z_{\overline{B}}$ *and* $Z_{III} \cong Z_B$*:*

$$p_{gauge}(l) + \frac{1}{2}\rho u^2(l) + \rho g\left(Z(l) - Z_I\right) = \rho g\left(Z_{III} - Z_I\right)$$

Then, by considering the geometric relation $\frac{Z(l) - Z_I}{L - l} = \frac{\left(Z_{III} - Z_I\right)}{L}$ *and* $p_{tot}(l) = p_{gauge}(l) + \frac{1}{2}\rho u^2(l)$*, the equation can be transformed into:*

$$p_{tot}(l) + \frac{L - l}{L}\rho g\left(Z_{III} - Z_I\right) = \rho g\left(Z_{III} - Z_I\right)$$

Thus:

$$p_{tot}(l) = \rho g\left(Z_{III} - Z_I\right) - \frac{L - l}{L}\rho g\left(Z_{III} - Z_I\right) = \frac{l}{L}\rho g\left(Z_{III} - Z_I\right)$$

8) In your opinion, could the dynamic pressure term be replaced by a constant? If yes, which geometrical characteristic must the penstock have?

Yes. For of a penstock with a constant section, the dynamic pressure term can be considered as constant, since the flow velocity u(l) is the same in every location l thanks to the principle of conservation of mass.

9) Let's now put some numerical values on these results. Considering a pipe diameter D = 3 m and using the values listed in Table 2, calculate the following values :

    a. The output power P in kW.

    b. The electricity generation in kWh for one year.

    c. The annual sales of the electricity (neglecting the energy losses in the generator).

Table 2. Specific values of the penstock

parameter	value
λ	0.0100
Annual productivity	50%
Electricity Unit Price	0.06 CHF/kWh

a. The distributed specific energy losses are:

$$gH_r = \lambda \frac{8L}{\pi^2 D^5} Q^2 = 0.01 \times \frac{8 \times 5400}{\pi^2 \times 3^5} \times 50^2 = 450.34\,\mathrm{J\,kg^{-1}}$$

Then, the output power P can be calculated as:

$$P = \eta \rho Q \left(g\left(Z_B - Z_{\bar{B}} \right) - gH_r \right) = 113'348\,\mathrm{kW}$$

b. The electricity generation, considering the generator efficiency as equal to 1, and knowing that the annual productivity is 50%, is:

$$EG = Annual\ Productivity \times 24h \times 365\ days \times P = 4.97 \times 10^8\,\mathrm{kWh}$$

c. The annual sales of electricity are thus:

$$Co_{electricity} = EG \times Elec.Unit\ Price = 29.79\,\text{Million of CHF}$$